Hayward's Primary School

Calculation Policy 2023

Hayward's Maths Vision

At Hayward's, we strive for all children to develop a positive attitude to Maths as an interesting and valuable subject; we do not want children to fear Maths yet have a belief that they can achieve highly.

We strive for all children to become confident when talking about their mathematics, using reasoning – apparatus, diagrams and explanations – to show their understanding; we have high expectations of children's use of mathematical language.

We strive for all children to understand that Maths is not only the quick recall or computation of number facts – although these are fundamental – but that it is the considered thought and application using what they know to work through a problem; we want children to feel confident when solving problems.

We strive for all children to develop a range of efficient strategies – their own tool kits – and make choices about how to use their maths; we want children to make decisions and explain them.

We strive for all children to feel challenged but enthused by Maths; we want every child to enjoy and succeed as Mathematicians, understanding its very important place in our world.

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately

- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Guidance for following the Calculation Policy

Written methods of calculations are based on mental strategies.

Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording.

Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency.

When teaching a new strategy, it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages.

Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Our Methodology

At Hayward's, we want children to become confident in choosing the most efficient methods to solve a calculation. In many cases, children will have more than one method "up their sleeve" to find the solution to a calculation and will be able to pick the best one.

For children to be able to work with the columnar methods, we want children to have a sound understanding of place value. Many calculations should be solved mentally and/or with jottings; formal column methods are for calculations where the digits are varied and include regrouping or exchanging.

Correct Terminology

Correct	**Avoid**
regrouping	carrying
exchanging	borrowing/stealing
ones	units
calculation / equation	sum
is equal to / the same as	equals
unknown	answer

Calculation policy: Addition and Subtraction

Key language for addition: sum, total, parts and wholes, plus, add, altogether, more, ‘is the same as’

Key language for subtraction: take away, subtract, find the difference, fewer, less than

Addition and subtraction are inverse operations. Right from the start, children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that 6 + [] = 10 and 10 - 6 = [] are seen as ways of expressing the same question. Children should be encouraged to use inverse operations to check their solutions. Part-Whole models and bar models are visuals that show the relationship. It is also important to relate addition and subtraction to place value.

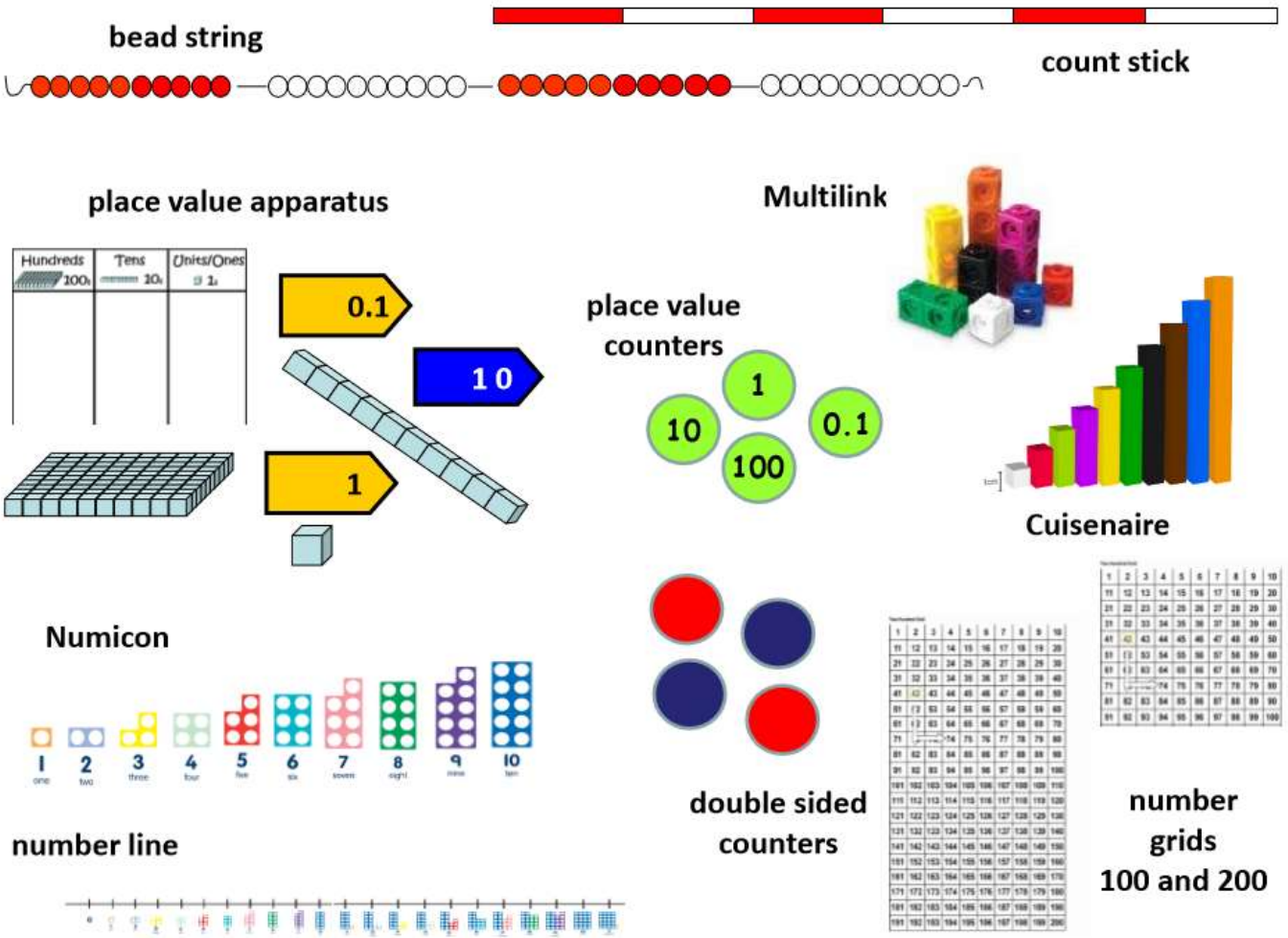
A digit’s true value should always be referenced.

Here is an example of language used to teach and talk through column addition:

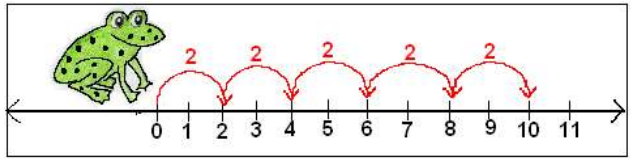
- Approximately, what will 245 + 378 be the same as? Let’s use rounding to estimate.
- I have 5 ones and I am adding 8 ones so now I have 13 ones which I can regroup or repartition as 1 ten and 3 ones.
- Next, I have 6 tens and I am adding 7 tens but I also have 1 ten from regrouping which makes 14 tens. I can regroup this as 1 hundred and 4 tens.
- Finally: 2 hundreds add 3 hundreds add 1 hundred from regrouping makes 6 hundreds.
- 243 + 368 is equal to 611.
- How could I write this as a subtraction?

Resources and Images for Addition and Subtraction

All learners should use resources and imagery to both develop and show understanding. Children should regularly draw pictures showing real life contexts for problems which they are solving. In order to help children remember certain strategies, certain references will be used throughout the school:



A spider may be used to help children understand the strategy of vertical jumps on a number square.



A frog may be used to help children understand the strategy of horizontal jumps on a numberline.

EYFS

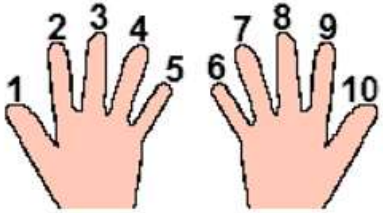
Age Band Statements (40-60 months)

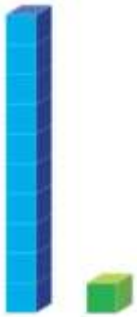
- Recognises numerals 1 to 5
- Recognise some numerals of personal significance
- Counts up to three or four objects by saying one number name for each item
- Counts actions or objects which cannot be moved
- Counts objects to 10 and beginning to count beyond 10
- Counts out up to six objects from a larger group
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects
- Counts an irregular arrangement of up to ten objects
- Estimates how many objects they can see and checks by counting them
- Uses the language of 'more' and 'fewer' to compare two sets of objects
- Finds the total number of items in two groups by counting all of them
- Says the number that is one more than a given number
- Finds one more or one less from a group of up to five objects, then ten objects
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting
- Records, using marks that they can interpret and explain

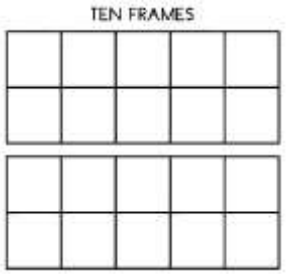
Early Learning

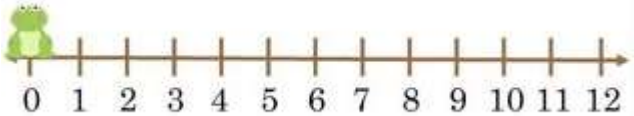
Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Apparatus is key in helping children to achieve the age band statements. Children should be encouraged to show their understanding through the full range of apparatus. Children should also be encouraged to draw their own pictures to help to achieve and show their understanding of the statements.







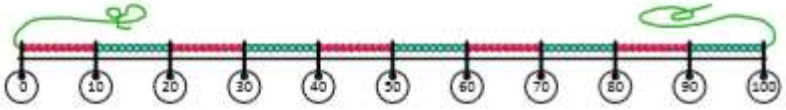


Year 1

Addition

Using place value

Count on in ones/counting in tens, *e.g. knowing 45 + 1 or 45 + 10 without counting on in ones.*



45 + ☐ = 50 65 + ☐ = 70

85 + ☐ = 90

Counting on

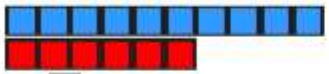
Count on in ones, *e.g. 11 + 2 =* and *7 + 4 =*

Count on in tens, *e.g. 45 + 20 as 45, 55, 65*

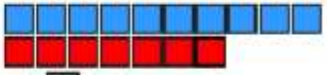
Using number facts

'Story' of 4, 5, 6, 7, 8 and 9, *e.g. 7 = 7 + 0 or 6 + 1 or 5 + 2 or 4 + 3.*

Number bonds to 10, *e.g. 5 + 5 , 6 + 4, 7 + 3, 8 + 2, 9 + 1, 10 + 0.*



6 + ☐ = 10



7 + ☐ = 10

5 + ☐ = 10

9 + ☐ = 10

Patterns using known facts, e.g. 4 + 3 = 7 so we know 24 + 3, 44 + 3, 74 + 3, etc.

Bead strings and 1-100 number grid help counting on/back in tens.

Missing number sentences, 3 + [] = 7, link addition and subtraction.

Subtraction

Using place value

Count back in 1s/Count back in 10s.

Say one less than any number to 100.

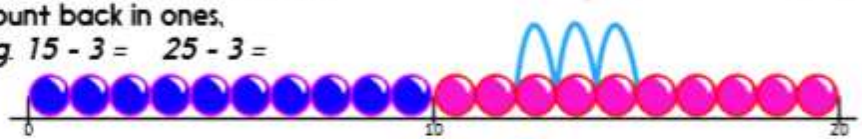
Say 10 less <u>without</u> counting back in ones.

33 - 10 = 23

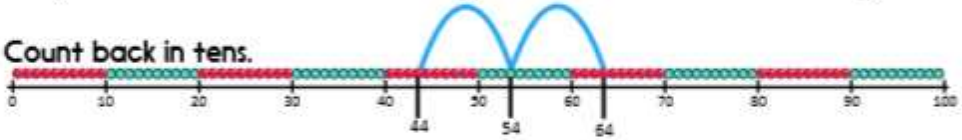
Subtracting by taking away

Count back in ones,

e.g. 15 - 3 = 25 - 3 =



Count back in tens.

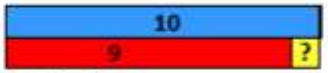


Using number facts

'Story' of 4, 5, 6, 7, 8 and 9, *e.g. 7 - 1 = 6, 7 - 2 = 5, 7 - 3 = 4, etc.*

Number bonds to 10, *e.g. 10 - 1 = 0, 10 - 2 = 8, 10 - 3 = 7, etc.*

10 - ☐ = 7



10 - ☐ = 9

Patterns using known facts,

e.g. 10 - 7 = 3 so we know 30 - 7 = ?

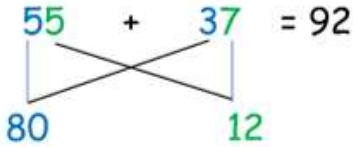
Year 2

Addition

Using place value

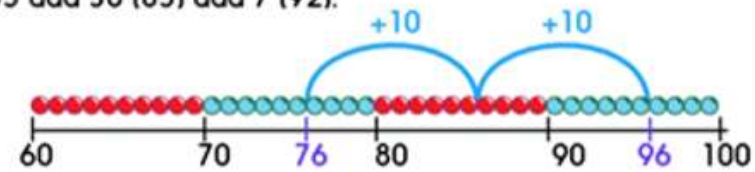
Know 1 more or 10 more than any number, e.g. 1 more than 67 or 10 more than 85.

Partitioning, e.g. 55 + 37 as 50 + 30 and 5 + 7 finally combining the two totals: 80 + 12.



Counting on

Add ten and mupltiples of ten, e.g. 76 + 20 as 76 , 86, 96 or in one hop 76 + 20. Add two 2-digit numbers by counting on in tens and then in ones, e.g. 55 + 37 as 55 add 30 (85) add 7 (92).



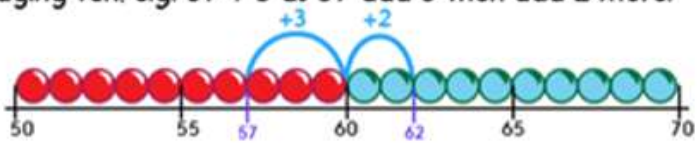
Add near multiples, e.g. 46 + 19 or 63 + 21.

Using number facts

Know pairs of numbers which make the numbers up to and including 10, e.g. *8 = 4&4, 3&5, 2&6, 1&7 and 10 = 5&5, 4&6, 3&7, 2&8, 1&9, 0&10.*

Patterns of known facts, *e.g. 6 + 3 = 9, so we know 36 + 3 = 39, 66 + 3 = 69, 53 + 6 = 59.*

Bridging ten, *e.g. 57 + 5 as 57 add 3 then add 2 more.*



Adding three or more single-digit numbers, spotting bonds to 10 or doubles, *e.g. 6 + 7 + 4 + 2 as 10 + 7 + 2.*

Bead strings and 1-100 number grid help counting on/back in tens.

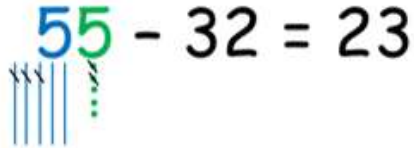
Missing number sentences, 3 + [] = 7, link addition and subtraction.

Subtraction

Using place value

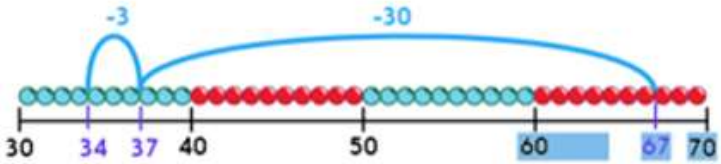
Know 1 less or 10 less than any number, e.g. 1 less than 74 or 10 less than 82.

Partitioning, e.g. 55 - 32 as 50 - 30 and 5 - 2 combining the answers: 20 + 3.



Taking away

Subtract ten and multiples of ten, *e.g. 76 - 20 as 76, 66, 56 or in one hop 76 - 20 = 56.* Subtract two 2-digit numbers by counting back in tens then in ones, *e.g. 67 - 33 as 67 subtract 30 (37) then count back 3 (34).*



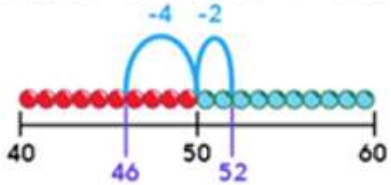
Subtracting near multiples, *e.g. 74 - 21 or 57 - 19.*

Using number facts

Know pairs of numbers which make the numbers up to and including 10, *e.g. 10 - 6 = 4, 8 - 3 = 5, 5 - 2 = 3, etc.*

Patterns of known facts, *e.g. 9 - 6 = 3, so we know 39 - 6 = 33, 69 - 6 = 63, 89 - 6 = 83.*

Bridge ten, *e.g. 52 - 6 as 52 subtract 2 then subtract 4 more.*



Counting up

Find a difference between two numbers on a line, *e.g. 51 - 47.*

Year 3

Addition

Using place value

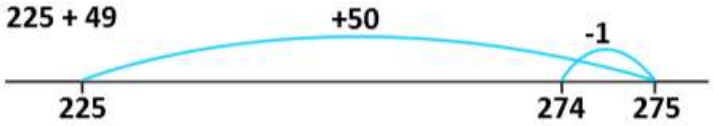
Count in hundreds, e.g. knowing 475 + 200 as 475, 575, 675.

Add multiples of 10, 100 and £1,
e.g. 746 + 200 or 746 + 40 or
£6.34 + £5 as £6 + £5 and 34p.

Partitioning, e.g. 68 + 74 as 60 + 70 and 8 + 4 and combine the totals: 130 + 12 = 142
Or £8.50 + £3.70 as £8 + £3 and 50p + 70p and combine: £11 + £1.20.

Counting on

Add 2-digit numbers by adding the multiple of ten then the ones,
e.g. 67 + 55 as 67 add 50 (117) add 5 (122).
Add near multiples of 10 and 100, e.g. 67 + 39 or 364 + 199.



Count on from 3-digit nos, e.g. 247 + 34 as 247 + 30 (277) then 277 + 4 = 281.

Using number facts

Number bonds to 100, e.g. 35 + 65, 46 + 54, 73 + 27, etc.

100	
65	35

Add to next ten and next hundred, e.g. 176 + 4 = 180, 435 + 65 = 500, etc.

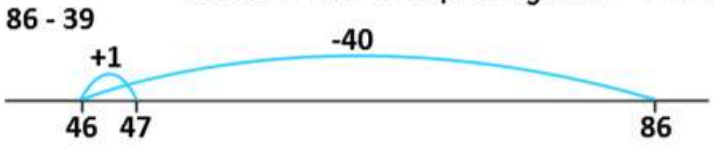
Subtraction is both taking away and - importantly - difference.

We no longer count in 1s but use PV and number facts.

Subtraction

Taking away

Use place value to subtract, e.g. 358 – 300 or 348 – 40 or 348 – 8.
Taking away multiples of 10, 100 and £1, e.g. 476 – 40 = 436,
476 – 300 = 176, £4.76 - £2 = £2.76.
Partitioning, e.g. 68 – 42 as 60 – 40 and 8 – 2 or
£6.84 - £2.40 as £6 - £2 and 80p – 40p.
Count back in hundreds, tens and then ones,
e.g. 763 – 121 as 763 – 100 (663) then subtract 20 (643) then subtract 1 (642).
Subtract near multiples, e.g. 648 – 199 or 86 – 39.



Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. 121 – 87.

Using number facts

Number bonds to 100, e.g. 100 – 35 = 65, 100 – 48 = 52, etc.

100	
48	?

Year 3

Written Addition

Written methods

Build on partitioning to develop expanded column addition with two 3-digit numbers.

	400	60	6
+	300	50	8
	700	110	14

Expanded column addition with 'carrying'.

	400	60	6
+	300	50	8
	800	20	4
	100	10	

Compact column addition with two or more 3-digit numbers or towers of 2-digit numbers.

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \hline 1\ \ 1 \end{array}$$

Compact column addition with 3-digit numbers

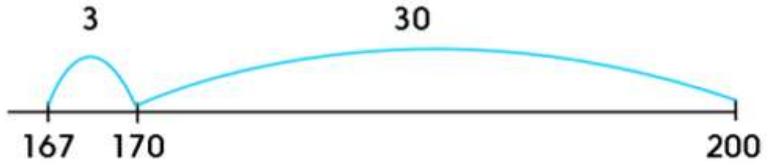
Recognise fractions which add to 1, e.g. $\frac{1}{4} + \frac{3}{4} = 1$ or $\frac{2}{5} + \frac{3}{5} = 1$

Counting up subtraction is a crucial mental strategy.

Important to see the visual image of fractions totalling one whole.

Written Subtraction

Develop counting up subtraction.



Use counting up subtraction to find change from £1 and £10.

£3 + 16p = £3.16

Expanded column subtraction, moving onto exchanging when ready:

258 - 73 = 185

$$\begin{array}{r} 200 + 50 + 8 \\ -\quad 70 + 3 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 100 + 150 + 8 \\ -\quad 70 + 3 \\ \hline 100 + \quad 80 + 5 \end{array} = 185$$

You might replace the **+ sign** with the word **'and'** to avoid confusion.

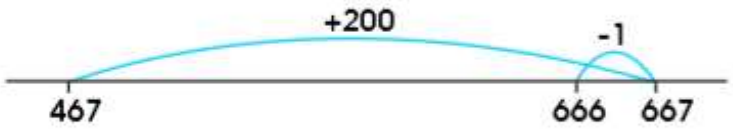
Year 4

Addition

Using place value
Count in thousands, e.g. knowing 475 + 200 as 475, 575, 675.
Partitioning, e.g. 746 + 203 as 700 + 200 and 46 + 3
or 134 + 707 as 130 + 700 and 4 + 7.

PV and number facts are central to mental strategies.

Counting on
Add 2-digit numbers by adding the multiple of ten then the ones, e.g. 67 + 55 as 67 add 50 (117) add 5 (122).
Add near multiples of 10, 100 and 1000, e.g. 467 + 199 or 3462 + 2999.



Count on to add 3-digit numbers and money, e.g. 463 + 124 as
463 + 100 (563) + 20 (583) + 4 = 587 or £4.67 + £5.30 as £9.67 add 30p.

Using number facts
Number bonds to 100 and to next multiple of 100,
e.g. 463 + 37, 1353 + 47.

Number bonds to £1 and to the next whole pound,
e.g. £3.45 + 55p.
Add to the next whole number, e.g. 4.6 + 0.4 or 7.2 + 0.8.

Subtraction

Taking away
Use place value to subtract, e.g. 4748 - 4000 or 4748 - 8, etc.

Take away multiples of 10, 100, 1000, £1, 10p or 0.1, e.g. 8392 - 50 or 6723 - 3000 or £3.74 - 30p or 5.6 - 0.2.

Partioning, e.g. £5.87 - £3.04 as £5 - £3 and 7p - 4p or 7493 - 2020 as 7000 - 2000 and 90 - 20.

Count back, e.g. 6482 - 1301 as 6482 - 1000, then - 300, then -1 (5181)
Subtract near multiples, e.g. 3522 - 1999 or £34.86 - £19.99.

Counting up
Find a difference between two numbers by counting up from the smaller to the larger, e.g. 506 - 387.

100 + 10 + 6 + 3 = **119**

Using number facts
Number bonds to 10, 100 and derived facts,
e.g. 100 - 76 = 24, 1.0 - 0.6 = 0.4.

Counting up is essential for money calculations and, later, decimals.

100	
76	24

Number bonds to £1 and £10,
e.g. £1.00 - 86p = 14p or £10 - £3.40 = £6.60.

Year 4

Written Addition

Build on expanded column addition to develop compact column addition with larger numbers.

	1000	400	60	8
+	4000	800	60	6
	6000	300	30	4
	1000	100	10	

Compact column addition with larger numbers.

	5	3	4	7
	2	2	8	6
+	1	4	9	5
	9	1	2	8
	1	2	1	

Use expanded and compact column addition to add amounts of money, e.g. £3.24 + £2.58.

£3	20p	4p	
£2	50p	8p	
£5	70p	12p	£5.82

	£3.24
+	£2.58
	£5.82
	1

Add like fractions, e.g. $\frac{3}{8} + \frac{2}{8} + \frac{1}{8}$

Expanded methods firm up a robust understanding of place value.

Stress that decimals and fractions are parts of a whole.

Written Subtraction

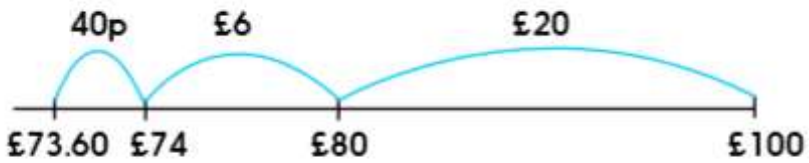
Expanded column subtraction.

	600	110	16
	~~700~~	~~20~~	~~6~~
-	300	50	8
	300	60	8

Begin to use column subtraction.

	6	11	16
	~~7~~	~~2~~	~~6~~
-	3	5	8
	3	6	8

Use counting up subtraction to find change from £10, £20, £50 and £100, e.g. £100 - £73.60.



£20 + £6 + 40p = £26.40

Subtract like fractions, e.g $\frac{7}{8} - \frac{2}{8} + \frac{5}{8}$

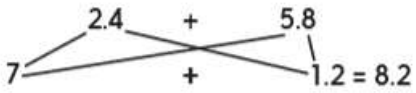
Year 5

Addition

Using place value

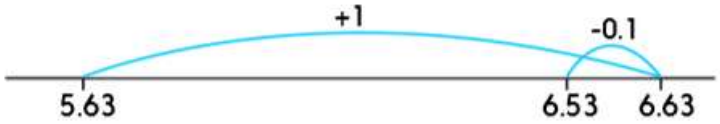
Count in 0.1s, 0.01s, e.g. knowing what 0.1 more than 0.51 is.

Partitioning, e.g. 2.4 + 5.8 as 2 + 5 and 0.4 + 0.8 and combine the totals: 7 + 1.2 = 8.2.



Counting on

Add two decimal numbers by adding the ones then the tenths/hundredths, e.g. 5.72 + 3.05 as 5.72 add 3 (8.72) then add 0.05 (8.77).
Add near multiples of 1, e.g. 6.34 + 0.99 or 5.63 + 0.9.

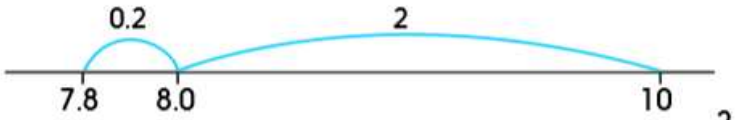


Count on from large numbers, e.g. 6834 + 3005 as 9834 + 5.

Using number facts

Number bonds to 1 and to the next whole number, e.g. 0.4 + 0.6 or 5.7 + 0.3.

Add to the next ten from a decimal number, e.g. 7.8 + 2.2 = 10.



2 + 0.2 = 2.2.

Subtracting by counting up is much less error prone.

Knowledge of number bonds underpins mental stratgegies.

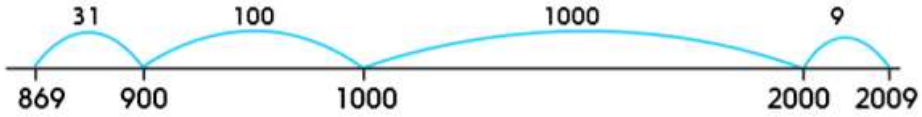
Subtraction

Taking away

Using place value to subtract decimals, e.g. 4.58 - 0.08 or 6.26 - 0.2, etc.
Take away multiples of powers of 10, e.g. 15, 672 - 300 or 4.82 - 2 or 2.71 - 0.5 or 4.68 - 0.02.
Partition or count back, e.g. 3964 – 1051 or 5.72 – 2.01.
Subtract near multiples, e.g. 86,456 – 9999 or 3.58 – 1.99.

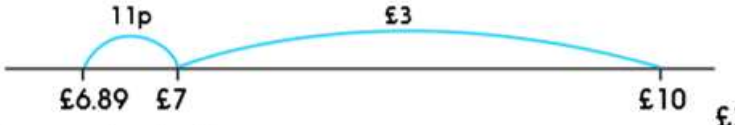
Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. 2009 - 869.



1000 + 100 + 31 + 9 – 1140

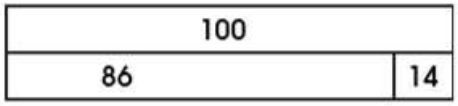
Find change using shopkeepers' addition, e.g. buy toy for £6.89 using £10.



£3 + 11p = £3.11

Using number facts

Derived facts from number bonds to 10 and 100, e.g. 2 – 0.45 using 45 + 55 = or 100 or 3.00 – 0.86 using 86 + 14 = 100.



Number bonds to £1, £10 and £100, e.g. £4.00 - £3.86p = 14p or £100 - £66 using 66 + 34 = £100.

Year 5

Written Addition

Expanded column addition for money leading to compact column addition for adding several amounts of money.

	£14	60p	4p	
	£28	70p	8p	
+	£12	20p	6p	
	£55	60p	8p	£55.68
	£1	10p		

Expanded version first embeds understanding of place value.

Compact column addition to add pairs of 5-digit numbers.

Continue to use column addition to add towers of several larger numbers.

Use compact addition to add decimal numbers with up to two places.

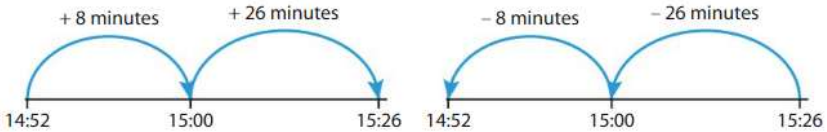
$$\begin{array}{r} 15.68 \\ +\ 27.86 \\ \hline 43.54 \\ \hline 1\ 1\ \ 1 \end{array}$$

Adding fractions with related denominators,

e.g. $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$

Equivalent fractions are the basis for + and - fractions.

Number lines are useful when working with time.
You don't have to think about the columns.



Written Subtraction

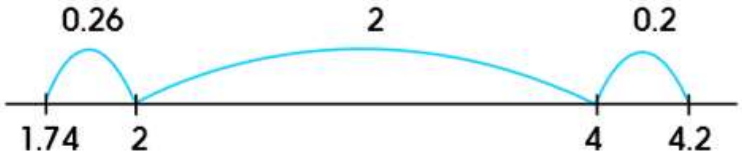
Compact column subtraction for numbers with up to 5 digits, e.g. 16,324 - 8516.

	0	15	13	1	14
-	~~1~~	~~6~~	~~3~~	~~2~~	~~4~~
		8	5	1	6
		7	8	0	8

Continue to use counting up subtraction for subtractions involving money, including finding change or, e.g. £50 - £28.76.

£21 + 24p = £21.24

Use counting up subtraction to subtract decimal numbers, e.g. 4.2 – 1.74.



2 + 0.26 + 0.2 = 2.46

Subtracting fractions with related denominators,

e.g. $1\frac{1}{4} - \frac{3}{8} = 1\frac{2}{8} - \frac{3}{8} = \frac{10}{8} - \frac{3}{8} = \frac{7}{8}$

Year 6

Addition

Using place value

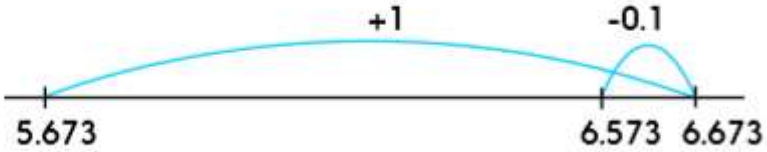
Count in 0.1s, 0.01s, 0.001s, e.g. knowing what 0.001 more than 6.725 is.
Partitioning, e.g. 9.54 + 3.25 as 9 + 3 and
0.5 + 0.2 and 0.04 + 0.05 to get 12.79.

10s	1s	.	0.1s 1/10s	0.01s 1/100s
	9	.	5	4
	3	.	2	5
1	2	.	7	9

Subtracting by counting up is much less error prone.

Counting on

Add two decimal numbers by adding the ones then the tenths/hundredths or thousandths, e.g. 6.314 + 3.006 as 6.314 add 3 (9.314) then add 0.006 (9.32).
Add near multiples of 1, e.g. 6.345 + 0.999 or 5.673 + 0.9.



Count on from large numbers, e.g. 16,375 + 12,003.

Using number facts

Number bonds to 1 and to the next multiple of 1,
e.g. 0.63 + 0.37 or 2.355 + 0.645.
Add to next ten, e.g. 4.62 + 0.38.

5	
4.62	?

Knowledge of number bonds underpins mental strategies.

Subtraction

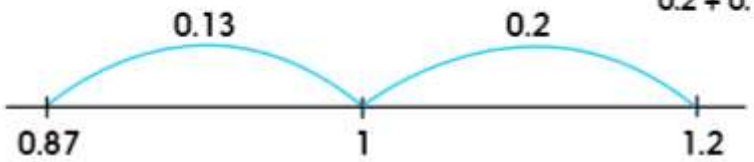
Taking away

Use place value to subtract decimals, e.g. 7.782 - 0.08 or 16.263 - 0.2, etc.
Take away multiples of powers of 10, e.g. 132,956 - 400 or 686,109 - 40,000 or 7.823 - 0.5.
Parition or count back, e.g. 3964 - 1051 or 5.72 - 2.01.
Subtract near multiples, e.g. 360,078 - 99,998 or 12.831 - 0.99.

Counting up

Count up to subtract numbers from multiples of 10, 100, 1000, 10,000
Find a difference between two decimal numbers by counting up from the smaller to the larger, e.g. 1.2 - 0.87.

0.2 + 0.13 = 0.33



Using number facts

Derived facts from number bonds to 10 and 100, e.g. 0.1 - 0.075 using 75 + 25 = 100 or 5 - 0.65 using 65 + 35 = 100.

Number bonds to £1, £10 and £100,
e.g. £7.00 - £4.37 or £100 - £66.20
using 20p + 80p = £1 and £67 + £33 = £100.

£100	
£67	£33

Year 6

Written Addition

Compact column addition for adding several large numbers and decimals with up to two places.

Compact column addition with money.

Children must be able to do expanded as well as compact to show understanding.

```
  154·75
+ 233·82
  388·57
  ------
     1
```

```
  21848
+  1523
  23371
  -----
    1 1
```

```
  £14.64
  £28.78
+ £12.26
  ------
  £55.68
  ------
```

Adding fractions with unlike denominators

$\frac{3}{4} + \frac{1}{3} = 1\frac{1}{12}$ or $2\frac{1}{4} + 1\frac{1}{3} = 3\frac{7}{12}$

$\frac{3}{4} + \frac{1}{3}$ $\qquad$ $\frac{9}{4} + \frac{4}{3}$

$= \frac{9}{12} + \frac{4}{12}$ $\qquad$ $= \frac{27}{12} + \frac{16}{12}$

$= \frac{13}{12}$ $\qquad$ $= \frac{43}{12}$

$= 1\frac{1}{12}$ $\qquad$ $= 3\frac{7}{12}$

Understanding equivalent fractions is absolutely key here.

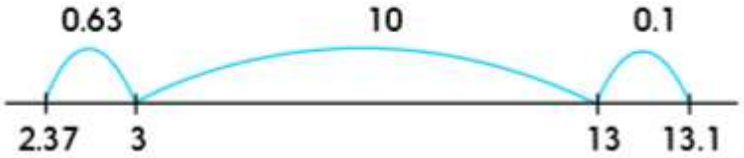
Written Subtraction

Compact column subtraction for large numbers.

```
  2 14   7 15
- 3 4 6 8 5
  1 6 4 5 8
  ---------
  1 8 2 2 7
```

Use counting up subtraction when dealing with money.
e.g. £100 – £78.56 or £45.23 - £27.57.

Use counting up subtraction to subtract decimal numbers.
e.g. 13.1 – 2.37.



10 + 0.63 + 0.1 = 10.73

Subtracting fractions with unlike denominators.

$1\frac{1}{4} - \frac{2}{3}$

$= \frac{5}{4} - \frac{2}{3}$

$= \frac{15}{12} - \frac{8}{12}$

$= \frac{7}{12}$

Calculation policy: Multiplication and Division

Key language for multiplication: multiply, times, factor, multiple, product, groups of, lots of, equal

Key language for division: divide, share, group, sharing, grouping, equal

Multiplication and division are inverse operations. Right from the start, children should be taught these as related operations. There are four number sentences (two using x and two using ÷) which can be written to express the relationship between 5 and 9 and 45. It is key to a good understanding of division that [] x 5 = 45 and 45 ÷ 5 = [] are seen as ways of expressing the same question. Like in addition and subtraction, equations can be written with the “equals” symbol not necessarily at the end of an equation: 5 = 45 ÷ 9.
The **product** is the number made when two (or more) numbers are multiplied together. Children should use the word **product** from Key Stage One. **Factors** are the numbers that can be multiplied to make a **product.** It is also important to relate multiplication and division to place value.

A digit’s true value should always be referenced.

Here are some examples of how linking back to place value can support the teaching of multiplication and division.

40 x 6 =

4 **ones** x 6 **=** 24 **ones** so that means that 4 **tens** x 6 = 24 **tens** which is **240.**

3600 ÷ 9 =

3600 is the same as 36 **hundreds.** 36 **hundreds** ÷ 9 = 4 **hundreds**

Progression

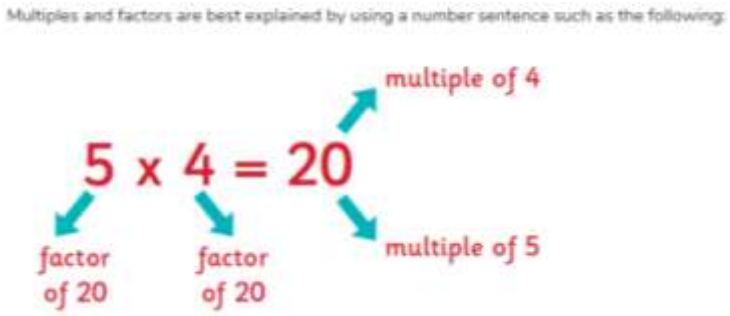
Step	
1	Equal groups (Size)
2	How many equal groups (Number)
3	Combining size and number of groups
4	Inefficiency of repeated addition → Multiplication
5	Notation and language of multiplication
6	Multiplication is Commutative
7	Securing times tables facts
8	Use variation to explore and deepen the concept (inc generalisation)
9	Written multiplication

Factors, Multiples, Primes, Squares and Cubes

Throughout primary school, children encounter special numbers. Children should know and use the vocabulary and be confident identifying and working with the numbers relevant to their year group.

A **multiple** is a number that can be divided by another number without a remainder.
A multiple of 3 is a number in the 3x table, a number in the pattern of counting in 3s.

A **factor** is a number which can be multiplied with another to produce another number.
Factors come in pairs, unless the number is a square number.



A **prime number** is a number which has only 2 factors, 1 and itself.
It does not appear in any multiplication tables other than its own.

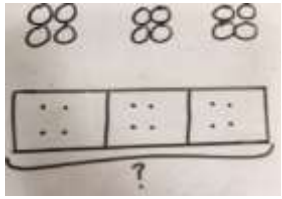
A **square number** is a number produced when a number is multiplied by itself.
3 x 3 = 9 so 9 is a square number. Square numbers have an odd number of factors for this reason.

A **cubed number** is a number produced when a number is multiplied by itself and itself again.
3 x 3 x 3 = 27 so 27 is a cubed number.

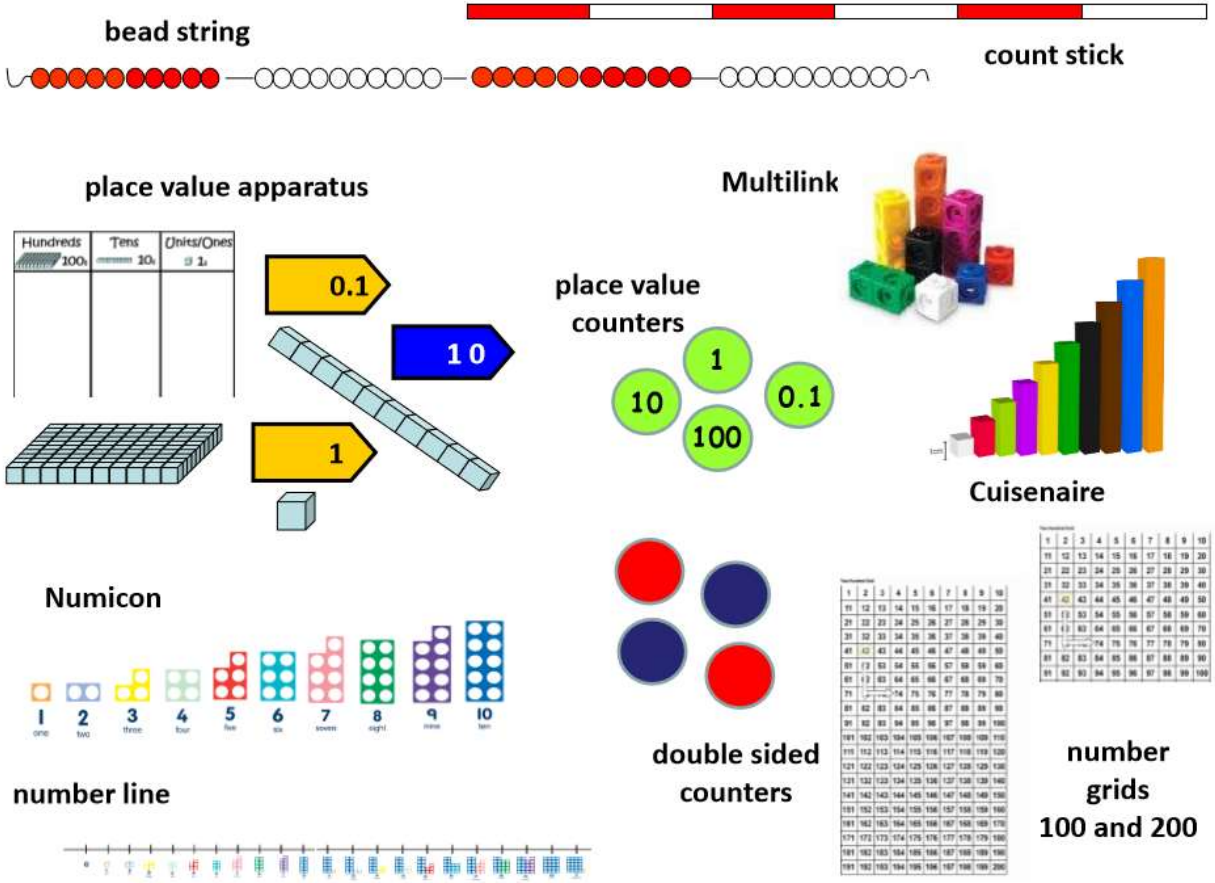
Resources and Images for Multiplication and Division

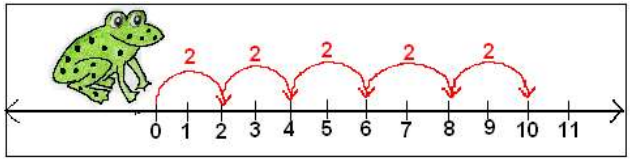
All learners should use resources and imagery to both develop and show understanding. Particularly in Key Stage One, where there are no formal methods for multiplication and division, children will be working with concrete objects and pictures. Children will record their work and show their methods by drawing pictures.

An important first step in the understanding of multiplication and division is understanding the concept of **equal groups.** Children will first use concrete objects and pictures to show their understanding of this.



Children will use objects and pictures to show their workings, especially in Key Stage One.



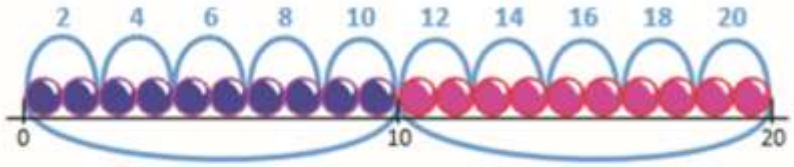


For “clever counting” (counting in steps) a frog may be used to help children understand the strategy of horizontal jumps on a numberline.

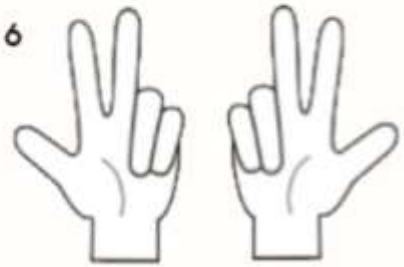
Year 1

Multiplication

Counting in steps ('Clever' counting)
Count in 2s and 10s.

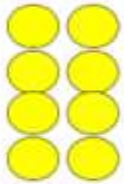


Doubling and halving
Find doubles to double 6 using fingers.



Grouping
Begin to use visual and concrete arrays and 'sets of' objects

There are 5 balloons in a group.
There are 4 groups.
5 + 5 + 5 + 5 = 20
4 lots of 5 = 20



2 counters are in a row.
There are 4 rows.
How many counters are there?
2+2+2+2 = 8

Children should be **unitising** to find totals: counting in groups, working towards using known facts.

Addition: (3)+(3)+(3)+(3)+(3)=(15)blocks

Multiplication: (3)×(5)=(15)blocks

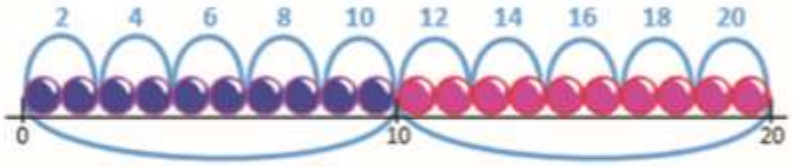
Some children may begin to use the multiplication symbol.

'Clever' counting is an excellent basis for multiplication and division.

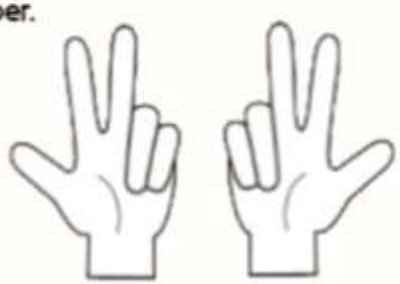
Division must be presented as the inverse of multiplication (grouping).

Division

Couting in steps ('Clever' counting)
Count in 2s, and 10s.

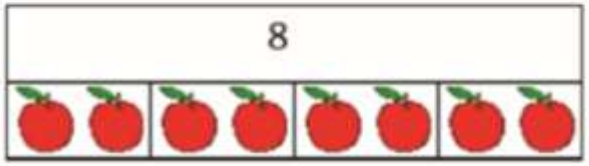


Doubling and halving
Find half of even numbers up to 12 including realising that it is hard to halve an odd number.



Grouping
Begin to use visual and concrete arrays and 'sets of' objects to find the answers to *'how many towers of 3 can I make with 12 cubes?'*

Sharing
Begin to find half of a quantity using sharing, *e.g half of 16 cubes by giving one each repeatedly to two children.*

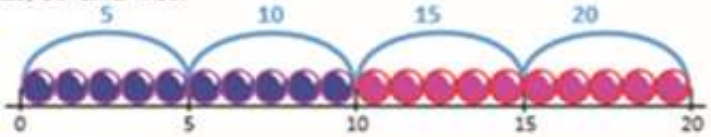


Some children may begin to use the division symbol.

Year 2

Multiplication

Counting in steps ('Clever' counting)
Count in 2s, 5s and 10s.



Begin to count in 3s.

Doubling and halving
Begin to know doubles of multiples of 5 to 100.
e.g. double 35 is 70.

Grouping
Use stem sentences:
There are ____ pencils in each packet. There are ____ packets.
10 + 10 + 10 = 10 x 3 = 30

Use arrays to explore the commutative nature of multiplication. 5 + 5 + 5 = 5 x 3 = 3 x 5.
3 lots/groups of 5 is equal to 5 lots/groups of 3.

Use number facts
Know doubles to double 20

Double 7 = 14

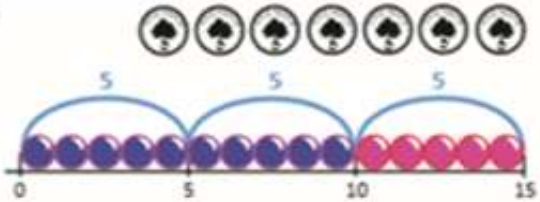
Start learning 2x, 5x, 10x tables, relating these to 'Clever counting' in 2s, 5s, and 10s, *e.g. 5 x 10 = 50, and 10, 20, 30, 40, 50 is five steps in the tens count.*

'Clever' counting is an excellent basis for multiplication and division.

Division, grouping, is the inverse of multiplication.

Division

Counting in steps ('Clever' counting)
Count in 2s, 5s and 10s



Doubling and halving
Find half of numbers up to 40, including realising that half of an odd number gives a remainder of 1 or an answer containing a ½.
Begin to know half of multiples of 10 to 100, *e.g. half of 70 is 35.*

Grouping
Relate division to multiplication by using arrays, objects and pictures. *e.g I have 20 sausages and I put 4 on each plate, many plates do I need? How many 4s are in 20?*

Sharing
Relate division to the sharing of a total into a given number of equal groups. Explore with concrete objects, materials, pictures and arrays.
e.g the teacher has 30 pencils to share between 3 children. How many pencils do they get each?
The children will share the pencils equally between 3 groups.

Using number facts
Know halves of even numbers to 24.
Know 2x, 5x and 10x division facts.
Begin to know 3x division facts.

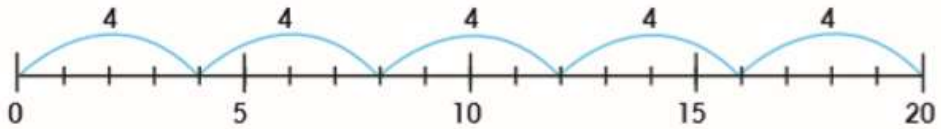
half of 20 is...

20	
?	?

Year 3

Mental Multiplication

Counting in steps ('Clever' counting)
Count in 2s, 3s, 4s, 5s, 8s and 10s, e.g. colour the multiples on a 1-100 grid or use hops along a landmarked line.



Doubling and halving
Find doubles to double 50 using partitioning.
Use doubling as a strategy in multiplying by 2,
e.g. 18 x 2 is double 18 (36).

48
80 + 16 = 96

Grouping
Recognise that multiplication is commutative, e.g. 4 x 8 = 8 x 4.
Multiply multiples of 10 by single-digit numbers, e.g. 30 x 8 = 240.
Multiply friendly 2-digit numbers by single-digit numbers, e.g. 13 x 4.

Use dienes to show and explore the relationship between multiplication:
3 ones x 8 = 24 ones so 30 x 8 = 240 because:
3 tens x 8 = 24 tens which is 240 ones, so 240.

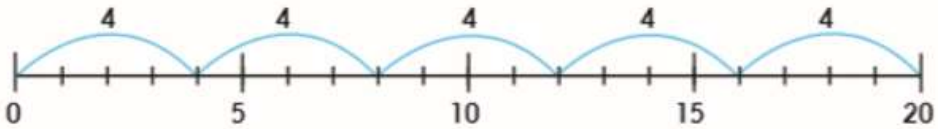
Using number facts
Know doubles to 20 and doubles of multiples of 5 to 100, e.g. double 45 is 90.
Know doubles of multiples of 5 to 100, e.g. double 85 is 170.
Know 2x, 3x, 4x, 5x, 8x, 10x tables facts.

Doubling and halving form the basis of mental x & ÷ strategies.

Number facts must be memorised and used on a daily basis.

Mental Division

Counting in steps ('Clever' counting)
Count in 2s, 3s, 4s, 5s, 9s and 10s by colouring numbers on the 1-100 grid or using a landmarked line.



Doubling and halving
Find half of even numbers to 100 using partitioning.
Use halving as a strategy in dividing by 2,
e.g. 36 ÷ 2 is half of 36.

36
15 + 3 = 18

Grouping
Recognise that division is not commutative, e.g. 16 ÷ 8 does not equal 8 ÷ 16.
Relate division to multiplications 'with holes in', e.g. ☐ x 5 = 30 is the same calculation as 30 ÷ 5 = ? thus we can count in in 5s to find the answer.
Divide multiples of 10 by single-digit numbers, e.g. 240 ÷ 8 = 30.

Using number facts
Know halves of even numbers to 40.

28	
?	?

Know halves of multiples of 10 to 200, e.g. half of 170 is 85.
Know 2x, 3x, 4x, 5x, 8x, 10x division facts.
Use divison facts to find unit and simple non-unit fractions of amounts within the times tables, e.g. ¾ of 48 is 3 x (48 ÷ 4).

Year 3

Written Multiplication

Using arrays to build understanding and make connections

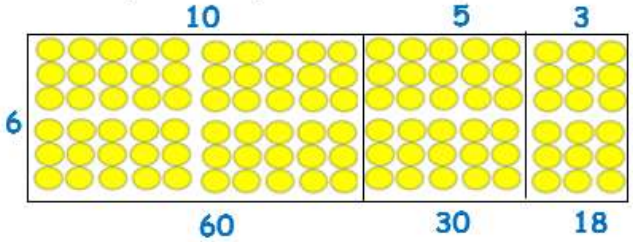
Use arrays to help children understand the relationships between calculations.

2 x 3 = 6 4 x 3 = 12

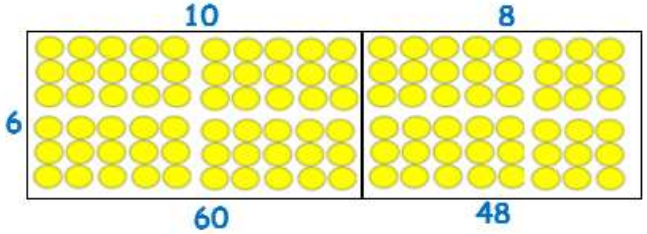
Encourage the exploration of arrays to make connections – that 4 x 3 is double 2 x 3 so the product is also double.
What would 8 x 3 be? Why?
What would 16 x 3 be? Why?

Progression towards grid multiplication

Use arrays to help children understand what the grid method is.



Children can explore partitioning the number in different ways, helping with their understanding of how multiplication tables link together.



Lead towards partitioning into the tens and the ones as in the abstract grid method.

Build on partitioning to develop grid multiplication.

x	20	3	=
4	80	12	92

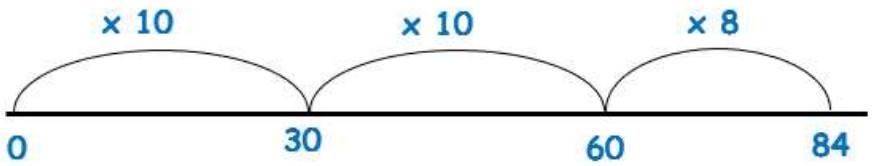
Written Division

Connect division to multiplication to solve division calculations

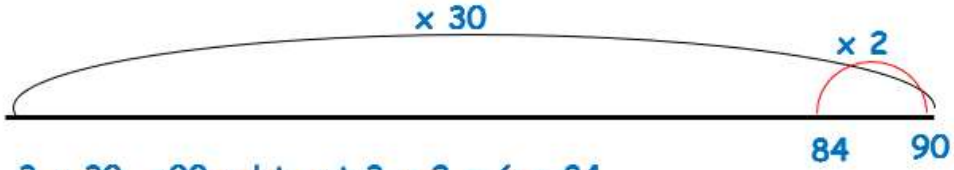
84 ÷ 3 = How many 3s in 84?

Use known facts, with numberline to reinforce understanding.

3 x 10 =30
3 x 10 =30
3 x 8 =24
3 x 28 =84 so 84 ÷ 3 = 28



Children can explore with different numberlines to find different ways of solving calculations.



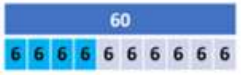
3 x 30 =90 subtract 3 x 2 = 6 = 84

Connect fractions to division

Understand that fractions relate to division – that tenths is dividing by 10, quarters is dividing by 4 etc

Children will use division methods to find fractions of small amounts:

$\frac{4}{10}$ of 60 = 60 ÷ 10 x 4



Year 4

Mental Multiplication

Counting in steps (sequences)
Count in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s, 12s, 25s, 50s, 100s and 1000s.

Doubling and halving
Find doubles to double 100 and beyond using partitioning,
e.g. double 226.

226
400 + 40 + 12 = 452

Begin to double amounts of money,
e.g. £3.50 doubled is £7.
Use doubling as a strategy in multiplying by 2, 4 and 8,
e.g. 34 x 4 = double 34 (68) doubled again (136).

Grouping
Use partitioning to multiply 2-digit numbers by single-digit numbers.
Multiply multiples of 100 by single-digit numbers using tables facts,
e.g. 400 x 8 = 3200.
Use dienes to show the relationship with place value.
4 ones x 8 = 32 ones so 400 x 8 = 3200 because:
4 hundreds x 8 = 32 hundreds which is 3200 ones, so 3200.
Use the abstract grid method to show multiplying by rounding
e.g. 7 x 19 = 7 x 20 - 7

19 20
7
7 x 20 = 140
140 - 7 = 133

Using number facts
Know times tables up to 12 x 12.

Facility in doubling and halving is key for mental x and ÷ strategies.

Mental Division

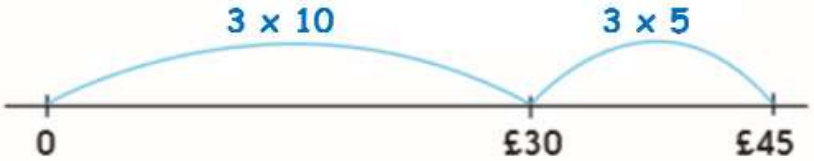
Counting in steps (sequences)
Count in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 11s, 12s, 25s, 50s, 100s and 1000s.

Doubling and halving
Find halves of even numbers to 200 and beyond using partitioning.

344	
172	172

Begin to half amounts of money, e.g. £9 halved is £4.50.
Use halving as a strategy in dividing by 2, 4 and 8, e.g. 164 ÷ 4 is half of 164 (82) halved again (41).

Grouping
Use multiples of 10 times the divisor to divide by numbers <9 above the tables facts, e.g. 45 ÷ 3.



Divide multiples of 100 by single-digit numbers using division facts,
e.g. 3200 ÷ 8 = 4000.

Using number facts
Know times tables up to 12 x 12 and all related division facts.
Use division facts to find unit and non-unit fractions of amounts within the times tables, e.g. 7/8 of 56 is 7 x (56 ÷ 8).

Year 4

Written Multiplication

Use grid multiplication to multipy 3-digit by 1-digit numbers.

x	200	50	3
6	1200	300	18

= 1518

If children understand place value they can develop fluency.

Formal Short Multiplication

Move from grid method to short multiplication to multiply 3-digit numbers by 1-digit numbers.

127 x 6 = 762

```
  1 2 7
  x   6
  ------
    4 2   (6x7)
+ 1 2 0   (6x20)
  6 0 0   (6x100)
  -------
  7 6 2
```

```
  1 2 7
  x   6
  -----
  7 6 2
  -----
  1 4
```

Use the expanded column method as a bridge from the grid method to the formal short method to demonstrate to children the place value. Use it as a teaching point towards children learning the short method.

Use the language of place value to ensure understanding.

6 x 7 ones is 42 ones, which is 4 tens and 2 ones, so the 4 is placed in the tens column and the 2 in the ones column.

6 x 2 tens is 12 tens, but we also have 4 tens already, making 16 tens. 16 tens is 1 hundred and 6 tens, so we place the 1 in the hundreds column.

6 x 1 hundred is 6 hundreds, plus the extra hundred we already have so we have 7 hundreds.

Ensure the digits that are regrouped are written under the line in the correct column, smaller than the actual digits in the calculation.

Written Division

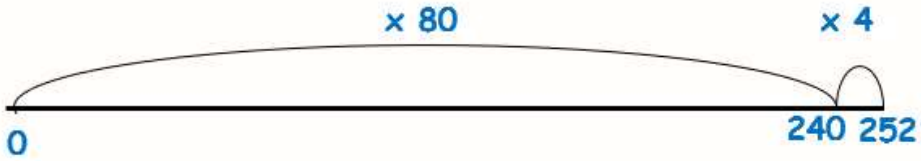
Connect division to multiplication to solve division calculations

Build on work from year 3 to continue to understand division as the inverse of multiplication, using known facts to solve calculations.

252 ÷ 3 = How many 3s in 252?

Use known facts and knowledge of partitioning, with a numberline to reinforce understanding.

3 x 80 =240
3 x 4 =12
3 x 84 = 252 so 252 ÷ 3 = 84



Introduce the formal division layout

Using numbers under 100, introduce children to formal division, at first using partitioning and known facts.

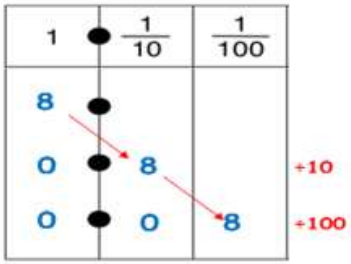
```
        10 + 4 = 14
7 ) 70 + 28
```

98 ÷ 7 = 14

```
      1 4
7 ) 9²8
```

Use a place value grid for dividing 1 digit numbers by 10 and 100

Identify the value of the digits in each number as ones, tenths and hundredths.



Year 5

Mental Multiplication

Doubling and halving

Double amounts of money using partitioning.
e.g. £6.73 doubled is double £6 (£12) plus double 73p (£1.46).

Use doubling and halving as a strategy in multiplying by 2, 4, 8, 5 and 20,
e.g. 58 x 5 = ½ of 58 (29) x 10 (290).

Partitioning remains a key skill throughout.

£6.73

£12 + £1.46 = £13.46

Grouping

Multiply decimals by 10, 100, 1000,
e.g. 3.4 x 100 = 340.

100s	10s	1s	.	0.1s
		3	.	4
3	4	0		

Use partitioning to multiply friendly 2-digit and 3-digit numbers by single-digit numbers, e.g. 402 x 6 as 400 x 6 (2400) and 2 x 6 (12).

Develop understanding of commutative law, associative law and distributive law to find "easy" routes to solve calculations
e.g. 5 x 42 x 4 = 42 x 20 = 42 x 10 x 2
25 x 84 = 84 x 100 ÷ 4 = 8400 ÷ 4

Learning times tables involves BOTH multiplication and division facts.

Using number facts

Use times tables facts up to 12 x 12 to multiply multiples of the multiplier,
6 x 4 = 24 so 0.6 x 4 = 6 x 4 ÷ 10 = 2.4

Mental Division

Doubling and halving

Halve amounts of money using partitioning, e.g. half of £14.84 as half of £14 and half of 84p.

£14.84

£7 + 42p = £7.42

Use doubling and halving as a strategy in dividing by 2, 4, 8, 5 and 20,
e.g. 115 ÷ 5 as double 115 (230) ÷ 10.

Grouping

Divide numbers by 10, 100, 1000 to obtain decimal answers with up to three places, e.g. 340 ÷ 100 = 3.4.

Use the 10th, 20th, 30th ... multiple of the divisor to divide friendly 2-digit and 3-digit numbers by single-digit numbers,
e.g. 186 ÷ 6 as 30 x 6 (180) and 1 x 6 (6).

Find unit and non-unit fractions of large amounts, e.g. ⅗ of 265 is 3 x (265 ÷ 5).

Use the bar model when representing fractions of numbers.

265				
53	53	53	53	53

Using number facts

Use division facts from the times tables up to 12 x 12 to divide mutiples of powers of ten of the divisor, e.g. 3600 ÷ 9 using 36 ÷ 9.

Year 5

Written Multiplication

Short multiplication of 2-digit, 3-digit and 4-digit numbers by 1-digit numbers.

```
  1 2 7        £1.2 7
  x   6          x   6
  7 6 2        £7.6 2
  1 4          1 4
```

Children may use column multiplication to solve money problems - though encourage children to use jottings and mental methods. Be clear on the place value of pounds and pence.

Multiply a 2-digit number by a 2-digit number, progressing to the multiplication of 2-digit number by a 3-digit and 4-digit number.

```
   23
 × 14
  92   (23 × 4)
  1
 230   (23 × 10)
 322
 1
```

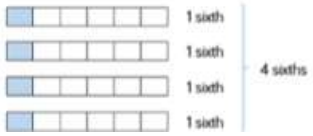
```
    3250
 ×    26
   19500
  13
+ 65000
  84500
  1
```

Ensure that children have a clear understanding of why the zero - place holder – is needed. Children can use grid multiplication to check their solutions; being confident in both methods is great.

Multiplying fractions

Children should connect multiplying proper and mixed fractions by a whole number to repeated addition and use visuals to aid understanding.

$$\frac{1}{6} \times 4 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$



The closer division is linked to multiplication the better.

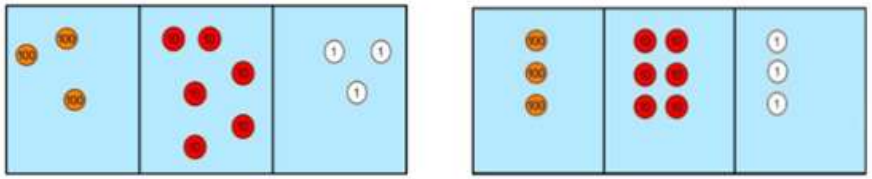
Visual images are essential to multiplying and dividing fractions.

Written Division

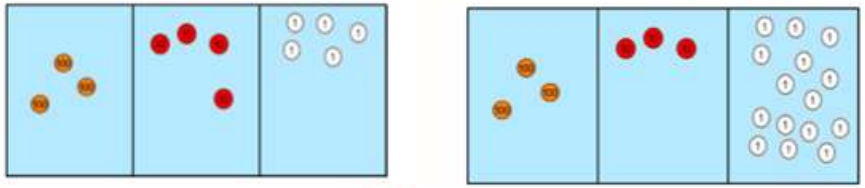
Short divison of 3-digit and 4-digit numbers by single-digit numbers.

```
      1 2 6 4
6 | 7 15 38 24
```

Children who are struggling to grasp the method from year 4 may use place value counters to aid understanding.



163 ÷ 3 is seeing "how many groups of 3" are in 163.
Children can use counters to make groups of 3.
163 ÷ 3 = 121.
Children can use counters to help understand exchanging where a full group cannot be made. 345 ÷ 3 =



```
      1 1 5
3 | 3 4 15
```

432 ÷ 5 = 86 r2

```
      8 6 r 2
5 ) 4 3 32
```

Children will learn to express the remainder as a fraction – the remainder divided by the divisor.

$$432 \div 5 = 86\frac{2}{5}$$

Year 6

Mental Multiplication

Doubling and halving

Double decimal numbers with up to 2-places using partitioning, e.g. 36.73 doubled is double 36 (72) plus double 0.73 (1.46).

36.73

72 + 1.46 = 73.46

Use doubling and halving as strategies in mental multiplication.

Grouping

Use partioning as a strategy in mental multiplication, as appropriate, e.g. 3060 x 4 as (3000 x 4) + (60 x 4) or 8.4 x 8 as 8 x 8 (64) and 0.4 x 8 (3.2)

Use factors in mental multiplication,
e.g. 421 x 6 as 421 x 3 (1263) doubled (2526)
or 3.42 x 5 as half of 3.42 x 10.

Mutliply decimal numbers using near multiples by rounding,
e.g. 4.3 x 19 as 4.3 x 20 (86 – 4.3).

Using number facts

Use times tables facts up to 12 x 12 in mental multiplication of large numbers or numbers with up to two decimal places,
e.g. 6 x 4 = 24 and 0.06 x 4 = 0.24.

Understanding how to partition numbers underpins many calculation strategies.

Division as grouping, i.e. the inverse of multiplication, is a key concept.

Mental Division

Doubling and halving

Halve decimal numbers with up to 2-places using partitioning, e.g. half of 36.86 if half of 36 (18) plus half of 0.86 (0.43).

36.86

18 + 0.43 = 18.43

Use doubling and halving as strategies in mental division, e.g. 216 ÷ 4 is half of 216 (108) and half of 108 (54).

Grouping

Use 10th, 20th, 30th, ... or 100th, 200th, 300th ... multiples of the divisor to divide large numbers, e.g. 378 ÷ 9 as 40 x 9 = 360 and 2 x 9 = 18 so, the answer is 42.

378 ÷ 9 = ___ x 9 = 378 = 9 x ___ = 378
9 x 40 =360
9 x 2 = 18
9 x 42 =378 so 378 ÷ 9 = 42

Use test for divisibility, e.g. 135 divides by 3 as 1 + 3 + 5 = 9 and 9 is in the 3x table.

Using number facts

Use division facts from the multiplication tables to help divide decimal numbers by single digit numbers
e.g. 9.6 ÷ 8 = 96 ÷ 8 ÷ 10 = 12 ÷ 10 = 1.2

Year 6

Written Multiplication

Short multiplication of 2-digit, 3-digit and 4-digit numbers by 1-digit numbers.

```
  1 2 7
  x   6
  7 6 2
  1 4
```

Long multiplication of 2-digit, 3-digit and 4-digit numbers by 2-digit numbers.

```
    3250
  x   26
  19500
 13
+ 65000
  84500
 1
```

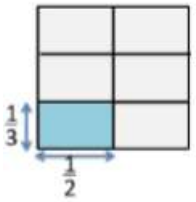
Short multiplication of decimal numbers using x 100 and ÷100, e.g. 13.72 x 6 as 1372 x 6 ÷100.

Short multiplication of money, e.g. £13.72 x 6 or £23.67 x 3.

```
  £1.2 7
     x   6
  £7.6 2
     1 4
```

Short versions of multiplication and division are more important and useful than the long versions.

Multiply proper and improper fractions by fractions, by using images to support understanding. Children should be encouraged to see the relationship between x and of.



$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$\frac{1}{3} of \frac{1}{2} = \frac{1}{6}$$

$$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

When children understand, they can apply to larger non-unit fractions.

Written Division

Short division of 3-digit and 4-digit numbers by single-digit numbers.

```
      1 2 6 4
  6 | 7 5 8 4
      1 3 2
```

Use knowledge of factor pairs to solve division calculations including 3-digit and 4-digit by a 2-digit number then use short division

7,848 ÷ 24

7848 ÷ 4 ÷ 6
7848 ÷ 2 ÷ 2 ÷ 6
2848 ÷ 2 ÷ 12

Long division of 3-digit and 4-digit numbers by two-digit numbers.

```
      200+50+1
  15 | 3 7 6 5
       3 0 0 0
       -------
         7 6 5
         7 5 0
         -----
           1 5
```

	x10	x100
15	0	0
30	0	0
45	0	0
60	0	0
75	0	0

Children write out the multiples and add a column for x10 and x100. They pick the closest number to that being divided, writing the number of groups along the top and subtracting from the total until complete.

Divide fractions by whole numbers by using $\frac{1}{6} \div 3$ imagery and context to support understanding.

$$\frac{1}{6} \div 3 = \frac{1}{6} \div \frac{3}{1} = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$$